**Testing Time Series Property (Non-Stationarity):**

Testing for stationarity is the first requirement for time series observations before going for estimation. Broadly a time series where mean, variance and covariance are time invariant is said to be stationary i.e. covariance or weakly stationary. The data which do not fulfill these properties are called non-stationary. A non-stationary process is also called as a unit root process. Moreover, the econometric models using non-stationary data are likely to violate the desirable statistical properties of the estimators or may give misleading inferences. Thus, it is necessary to test the stationarity of the time series before attempting any econometric exercise.

**Test of Stationarity or unit root Properties of Time series Data.**

A simple first order auto regressive process can be expressed by the following equation:

Y= (1)

Where {Y} is the stochastic process, and  are parameters and  is a random disturbance term with white noise properties.  is called drift or constant or intercept. The nature of the time series described in equation (1) depends on the values of the parameters. If 0 and <1, then Y follows a deterministic trend. The presence of auto regressive component will mean that there may be short run deviations, but the series will return to trend eventually. A series of this sort is known as a trend stationary (TS) process, as the residual from the regression when regressed on time will be stationary. If = 0, = 0 and = 1, the series is said to follow a simple random walk, a unit root process. If 0, =0 and =1, the series is said to follow a random walk, with drift. Any stochastic process, which becomes stationary after differencing once, is called a difference stationary (DS) process. For example, a simple random walk process is a DS process. Likewise, any time series, which becomes stationary after de-trending is called a (trend stationary) TS process.

In time series literature, there are both formal and informal tests of stationarity. The informal tests include time series plots and use of correlogram. Statistical packages use Box-Pierce Q-statistics and L-jung-Box Q-statistics for testing stationarity of series. These two statistics are based on autocorrelation coefficient of several lag lengths. The formal tests of non-stationarity are also known as unit root test or test of random walk series. These include Dickey-Fuller (DF), augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) test to check the presence of unit root in the data. These tests are necessary because the usual student’s ‘t’ test is inappropriate to test the null hypothesis, = 1 in equation 1.

***Dickey- Fuller and Augmented Dickey-Fuller Test***

The standard procedure of unit root test uses Dickey-Fuller test, which made the strict and unrealistic assumption of the error process being independently and identically distributed (iid) (0,) Gaussian processes. This DF methodology confines itself to pure Autoregressive Integrated Moving Average (ARIMA: 1, 0, 0) processes. Dickey and Fuller (1979) actually consider three different regression equations that can be used to test for the presence of unit root. The first is a pure random walk model, the second adds an intercept/drift term and the third include both a drift and linear time trend. The basic Dickey-Fuller (DF) test examines whether the value of the parameter is one (=1) in equation1. It means, whether the underlying first order difference equation has a unit root. Assuming the absence of trend term in equation 1 and rewriting it in a modified form we get

 (2)

where, 

The null hypothesis is that the {yt} process has a unit root. More generally, if the given time series follows a pth order auto regressive process [AR(p)] or even auto regressive moving average process [ARMA (p, q)] an extended Dickey-Fuller test called Augmented Dickey-Fuller (ADF) test is suggested. Specifically, if the original time series follows AR (p), it can be represented as

= (3)

After suitable mathematical manipulation, equation 3 can be rewritten as,

 (4)

where  and 

Equation 4 is also recommended if the residual sequence {} in equation 2 is not white noise, or when are auto-correlated. There are different forms of DF and ADF tests, which are possible by including trend terms in equation 2 and 4 and also excluding drift term, , from these equations. The DF test is a special case of ADF test when p = 1. To test the significance of  in equation 2 and 4, the usual student’s ‘t’ statistic critical values cannot be used. Initially, Dickey-Fuller and later Mac Kinnon have developed the appropriate test statistic known as statistic, and its critical value using Monte Carlo Simulations. The critical values of-statistic is made available under alternative assumptions of drift, trend, sample size and level of significance. They are abbreviated as  (no drift and no trend), (only drift) and (with both drift and trend). Dickey-Fuller have also provided the critical F-test values, known as and, for pair wise joint tests of significance for  and. Thus, the null hypothesis that = 0 can be rejected if the computed t-value for the coefficient  is greater than the critical value  -value in absolute magnitude. It has been shown that the same DF test critical values are valid for the ADF test as well.

***Phillips-Perron (PP) Test***

An important assumption of the DF test is that the error terms are uncorrelated or independently and identically distributed. The ADF test adjusts the DF test to take care of possible serial correlation in the error terms by adding the lagged difference terms of the regressand. Phillips and Perron use non parametric statistical methods to take care of the serial correlation in the error terms without adding lagged difference terms. Phillips and Perron (1988) developed a generalization of DF procedure that allows for fairly mild assumption concerning the distribution of the errors. The PP test considers two equations:

=+ …………… (5)

and = ……………. (6)

where T is the number of observation and is the disturbance term and E = 0 for all t. Phillips-Perron device statistics for the regression coefficients under the null hypothesis that the data are generated by

= ………………. (7)

The PP test can be applied to situations where the assumption of homoscedasticity and independently identically distributed error term may not be valid. Another advantage of PP test is that it can also be applied to frequency domain approach, which is more recent and an alternative to the usual time domain approach, to time series analysis. The PP test has been shown to follow the same critical values as that of DF test, but has greater power to reject the null hypothesis of unit root. However, the PP test seems to be biased towards rejecting the null hypothesis of a unit root, when the error series follows a negative moving average process. In such situations, it is better to use the ADF test, rather than the PP test.

***KPSS (Kwiatkowski-Phillips-Schmidt-Shin) Test***

KPSS is another test for checking the stationarity of a time series. Interestingly, the H0 and alternate hypothesis (H1) for the KPSS test are opposite of ADF test, which often creates confusion.

KPSS test of stationarity have defined the null hypothesis as the process is trend stationary, to an alternate hypothesis of a unit root series. A series is said to be trend stationary, if it has no unit root but exhibits a time trend. Once the trend is removed, the resulting series will be strict stationary. The KPSS test classifies a series as stationary i.e. the series can be strict stationary or trend stationary.

Strict stationary series implies, the mean, variance and covariance of a time series is not the function of time. It should be time invariant.

H0: The process is trend stationary.

H1: The series has a unit root (series is not stationary).

**How to interpret KPSS test results**

The output of the KPSS test contains 4 things:

1. The KPSS statistic
2. p-value
3. Number of lags used by the test
4. Critical values

The p-value reported by the test is the probability score based on which you can decide whether to reject the null hypothesis or not. If the p-value is less than a predefined alpha level (typically 0.05), we reject the null hypothesis.

The KPSS statistic is the actual test statistic that is computed while performing the test. For more information no the formula, the references mentioned at the end should help.

In order to reject the null hypothesis, the test statistic should be greater than the provided critical values. If it is in fact higher than the target critical value, then that should automatically reflect in a low p-value.

That is, if the p-value is less than 0.05, the kpss statistic will be greater than the 5% critical value.

Finally, the number of lags reported is the number of lags of the series that was actually used by the model equation of the kpss test.

**How is KPSS test different from ADF test**

A major difference between KPSS and ADF tests is the capability of the KPSS test to check for stationarity in the ‘presence of a deterministic trend’.

If you go back and read the definition of the KPSS test, it tests for stationarity of the series around a ‘deterministic trend. What that effectively means to us is, the test may not necessarily reject the null hypothesis (that the series is stationary) even if a series is steadily increasing or decreasing.

But what is a ‘deterministic trend’?

The word ‘deterministic’ implies the slope of the trend in the series does not change permanently. That is, even if the series goes through a shock, it tends to regain its original path.